

What is claimed is:

1 1. A method for reconstructing an image of a scattering medium, comprising:
2 directing energy into the scattering medium at a source location on the
3 scattering medium;
4 measuring the energy emerging from the scattering medium at a detector
5 location on the scattering medium;
6 selecting an initial guess of internal properties of the scattering medium;
7 predicting the energy emerging from the scattering medium using an
8 equation of radiative transfer, wherein the prediction is a function of the initial guess;
9 generating an objective function based on a comparison of the prediction
10 with the measurement;
11 generating a gradient of the objective function by a method of adjoint
12 differentiation;
13 modifying the initial guess of the properties of the scattering medium
14 based on the gradient of the objective function; and
15 generating an image representation of the internal properties of the
16 scattering medium.

1 2. The method according to claim 1, further comprising repeating the
2 predicting of the energy emerging from the scattering medium based on the modified
3 initial guess, generating the objective function and modifying the initial guess, until at
4 least one of a predetermined number of repetitions has occurred and the objective
5 function reaches a predetermined threshold.

1 3. The method according to claim 1, wherein the prediction depends on the
2 boundary conditions.

1 4. The method according to claim 3, wherein the boundary conditions
2 account for a refractive mismatch at an interface between the medium and at least one of
3 the detectors and source.

1 5. The method according to claim 1, wherein the prediction comprises an
2 iterative process producing intermediate results.

1 6. The method according to claim 5, wherein the intermediate results of the
2 prediction are stored.

1 7. The method according to claim 6, wherein generating the gradient of the
2 objective function by adjoint differences uses the intermediate results of the prediction.

1 8. The method according to claim 7, wherein generating the gradient
2 comprises stepping backward through the intermediate results of the prediction.

1 9. The method according to claim 1, wherein the equation of radiative
2 transfer is time independent.

1 10. The method according to claim 9, wherein the time independent equation
2 of radiative transfer is:

3
$$\omega \nabla \Psi(\mathbf{r}, \omega) + (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega) = S(\mathbf{r}, \omega) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega') d\omega'$$

4 where $\Psi(\mathbf{r}, \omega)$ is the radiance at the spatial position \mathbf{r} directed into a unit
5 solid angle ω , $S(\mathbf{r}, \omega)$ is the energy directed into the medium at spatial position \mathbf{r} into a
6 unit solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and
7 $p(\omega, \omega')$ is the scattering phase function.

1 11. The method according to claim 10, wherein the scattering phase function
2 is:

3
$$p(\cos \theta) = \frac{1 - g^2}{2(1 + g^2 - 2g \cos \theta)^{3/2}}$$

4 where θ is the angle between the two unit solid angles ω and ω' , and g is
5 the anisotropy factor.

1 12. The method according to claim 1, wherein the equation of radiative
2 transfer is time dependent.

1 13. The method according to claim 12, wherein the time dependent equation
2 of radiative transfer is:

3
$$\frac{1}{c} \frac{\partial \Psi(\mathbf{r}, \omega, t)}{\partial t} = S(\mathbf{r}, \omega, t) - \omega \cdot \nabla \Psi(\mathbf{r}, \omega, t) - (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega, t) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega', t) d\omega'$$

4 where $\Psi(\mathbf{r}, \omega, t)$ is the radiance at the spatial position \mathbf{r} directed into a unit
5 solid angle ω , $S(\mathbf{r}, \omega, t)$ is the energy directed into the medium at spatial position \mathbf{r} into a

6 unit solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and
7 $p(\omega, \omega')$ is the scattering phase function.

1 14. The method according to claim 13, wherein the scattering phase function
2 is:

3
$$p(\cos\theta) = \frac{1 - g^2}{2(1 + g^2 - 2g\cos\theta)^{3/2}}$$

4 where θ is the angle between the two unit solid angles ω and ω' , and g is
5 the anisotropy factor.

1 15. The method according to claim 1, wherein the properties include at least
2 one of a scattering coefficient, an absorption coefficient, an anisotropy factor, and a
3 scattering phase function.

1 16. The method according to claim 1, wherein the objective function is a
2 normalized comparison of the predicted energy and the measured energy

1 17. The method according to claim 1, wherein the objective function is based
2 on the normalized sum of the differences between the predicted energy and the measured
3 energy for each source detector pair, wherein a source detector pair is formed between
4 each source location and each detector location.

1 18. The method according to claim 1, wherein the objective function is:

2
$$\phi = \frac{1}{2} \sum_i^m (P_i - M_i)^2$$

3 where M_i represents the actual measurements and the P_i represents the
4 predicted measurements for each source defector pair i , m is the number of source
5 detector pairs, where a source detector pairs is formed between each source location and
6 each detector location.

1 19. The method according to claim 1, further comprising minimizing the
2 objective function.

1 20. The method according to claim 19, wherein minimizing the objective
2 function includes a one dimensional line search.

1 21. The method according to claim 20, wherein the one dimensional line
2 search is performed along a direction of the gradient of the objective function.

1 22. The method according to claim 20, wherein the one dimensional line
2 search is performed along a gradient-dependent direction.

1 23. The method according to claim 1, wherein the energy comprises near
2 infra-red energy.

1 24. The method according to claim 1, wherein the scattering medium contains
2 regions wherein the scattering coefficients are not substantially greater than the
3 absorption coefficients.

1 25. The method according to claim 1, wherein the scattering medium contains
2 a low scattering region embedded in a high scattering region.

1 26. The method according to claim 1, wherein the predicted energy is
2 determined using finite element methods.

1 27. The method according to claim 1, wherein the predicted energy is
2 determined using finite difference methods.

1 28. A method for imaging the spatial optical properties of tissue, comprising:
2 (a) directing energy into the scattering medium at a source location on
3 the tissue;
4 (b) measuring the energy emerging from the scattering medium at a
5 detector location on the tissue;
6 (c) selecting and initial guess of the spatial optical properties of the
7 tissue;
8 (d) predicting the energy emerging from the tissue using an equation
9 of radiative transfer in an iterative process, wherein the prediction is a function of the

10 initial guess and a refraction index mismatch at a boundary of the tissue, and the iterative
11 process generates a plurality of intermediate predictions;

12 (e) generating an objective function based on a normalized
13 comparison of the prediction with the measured energy emerging from the scattering
14 medium;

15 (f) generating a gradient of the objective function by adjoint
16 differentiation;

17 (g) modifying the initial guess of the spatial properties of the tissue
18 based on the gradient of the objective function;

19 (h) repeating steps (d) through (g) until at least one of a threshold of
20 modifications to the initial guess is reached and the objective function reaches a
21 threshold; and

22 (j) generating an image representation of the spatial optical properties
23 of the tissue.

1 29. A system for reconstructing an image of a scattering medium, comprising:
2 a source for directing energy into the scattering medium at source location on the
3 scattering medium;
4 a detector for measuring the energy emerging from the scattering medium at a
5 detector location on the scattering medium;
6 an initial guess of internal properties of the scattering medium;
7 means for predicting the energy emerging from the scattering medium using an
8 equation of radiative transfer, wherein the prediction is a function of the initial guess;

9 means for generating an objective function based on a comparison of the
10 prediction with the measurement;
11 means for generating a gradient of the objective function by a method of adjoint
12 differentiation;
13 means for modifying the initial guess of the properties of the scattering medium
14 based on the gradient of the objective function; and
15 means for generating an image representation of the internal properties of the
16 scattering medium.

1 30. The system according to claim 1, further comprising means for repeating
2 the predicting of the energy emerging from the scattering medium based on the modified
3 initial guess, generating the objective function and modifying the initial guess, until at
4 least one of a predetermined number of repetitions has occurred and the objective
5 function reaches a predetermined threshold.

1 31. The system according to claim 1, wherein the prediction depends on the
2 boundary conditions.

1 32. The system according to claim 31, wherein the boundary conditions
2 account for a refractive mismatch at an interface between the medium and at least one of
3 the detectors and source.

1 33. The system according to claim 1, wherein the prediction comprises an
2 iterative process producing intermediate results.

1 34. The system according to claim 33, wherein the intermediate results of the
2 prediction are stored.

1 35. The system according to claim 34, wherein generating the gradient of the
2 objective function by adjoint differences uses the intermediate results of the prediction.

1 36. The system according to claim 35, wherein generating the gradient
2 comprises stepping backward through the intermediate results of the prediction.

1 37. The system according to claim 1, wherein the equation of radiative
2 transfer is time independent.

1 38. The system according to claim 37, wherein the time independent equation
2 of radiative transfer is:

3
$$\omega \nabla \Psi(\mathbf{r}, \omega) + (\mu_a + \mu_s) \Psi(\mathbf{r}, \omega) = S(\mathbf{r}, \omega) + \mu_s \int_0^{2\pi} p(\omega, \omega') \Psi(\mathbf{r}, \omega') d\omega'$$

4 where $\Psi(\mathbf{r}, \omega)$ is the radiance at the spatial position \mathbf{r} directed into a unit solid
5 angle ω , $S(\mathbf{r}, \omega)$ is the energy directed into the medium at spatial position \mathbf{r} into a unit
6 solid angle ω , μ_s is the scattering coefficient, μ_a is the absorption coefficient and $p(\omega, \omega')$
7 is the scattering phase function.

3

4

5

2

2

3

4

2

3

1 46. The system according to claim 1, wherein the objective function is:

1 47. The system according to claim 1, further comprising minimizing the
2 objective function.

1 48. The system according to claim 47, wherein minimizing the objective
2 function includes a one dimensional line search.

1 49. The system according to claim 48, wherein the one dimensional line
2 search is performed along a direction of the gradient of the objective function.

1 50. The system according to claim 49, wherein the one dimensional line
2 search is performed along a gradient-dependent direction.

1 51. The system according to claim 50, wherein the energy comprises near
2 infra-red energy.

1 52. The system according to claim 1, wherein the scattering medium contains
2 regions wherein the scattering coefficients are not substantially greater than the
3 absorption coefficients.

1 53. The system according to claim 1, wherein the scattering medium contains
2 a low scattering region embedded in a high scattering region.

1 54. The system according to claim 1, wherein the predicted energy is
2 determined using finite element methods.

1 55. The system according to claim 1, wherein the predicted energy is
2 determined using finite difference methods.

1 56. A system for imaging the spatial distribution of optical properties of
2 tissue, comprising:

3 (a) a source for directing energy into the scattering medium at a source
4 location on the tissue;

5 (b) a detector for measuring the energy emerging from the scattering medium
6 at a detector location on the tissue;

7 (c) an initial guess of spatial optical properties of the tissue;

8 (d) means for predicting the energy emerging from the tissue using an
9 equation of radiative transfer in an iterative process, wherein the prediction is a function
10 of the initial guess and a refraction index mismatch at a boundary of the tissue, and the
11 iterative process generates a plurality of intermediate predictions;

12 (e) means for generating an objective function based on a normalized
13 comparison of the prediction with the measured energy emerging from the scattering
14 medium;

15 (f) means for generating a gradient of the objective function by adjoint
16 differentiation;

17 (g) means for modifying the initial guess of the spatial properties of the tissue
18 based on the gradient of the objective function;
19 (h) means for repeating steps (d) through (g) until at least one of a threshold
20 of modifications to the initial guess is reached and the objective function reaches a
21 threshold; and
22 (j) means for generating an image representation of the spatial optical
23 properties of the tissue.

1 57. Computer executable software code stored on a computer readable
2 medium, the code for reconstructing an image of a scattering medium, comprising:
3 code to direct energy into the scattering medium at a source location on
4 the scattering medium;
5 code to measure the energy emerging from the scattering medium at a
6 detector location on the scattering medium;
7 code to receive an initial guess of internal properties of the scattering
8 medium;
9 code to predict the energy emerging from the scattering medium using an
10 equation of radiative transfer, wherein the prediction is a function of the initial guess;
11 code to generate an objective function based on a comparison of the
12 prediction with the measurement;
13 code to generate a gradient of the objective function by a method of
14 adjoint differentiation;

15 code to modify the initial guess of the properties of the scattering medium
16 based on the gradient of the objective function; and
17 code to generate an image representation of the internal properties of the
18 scattering medium.

1 58. Computer executable software code stored on a computer readable
2 medium, the code for imaging the spatial distribution of optical properties of tissue,
3 comprising:

4 (a) code to direct energy into the scattering medium at a source location on the
5 tissue;

6 (b) code to measure the energy emerging from the scattering medium at a detector
7 location on the tissue;

8 (c) code to receive an initial guess of spatial optical properties of the tissue;

9 (d) code to predict the energy emerging from the tissue using an equation of
10 radiative transfer in an iterative process, wherein the prediction is a function of the initial
11 guess and a refraction index mismatch at a boundary of the tissue, and the iterative
12 process generates a plurality of intermediate predictions;

13 (e) code to generate an objective function based on a normalized comparison of
14 the prediction with the measured energy emerging from the scattering medium;

15 (f) code to generate a gradient of the objective function by adjoint differentiation;

16 (g) code to modify the initial guess of the spatial properties of the tissue based on
17 the gradient of the objective function;

18 (h) code to repeat steps (d) through (g) until at least one of a threshold of
19 modifications to the initial guess is reached and the objective function reaches a
20 threshold; and
21 (j) code to generate an image representation of the spatial optical properties of the
22 tissue.

1 59. A computer readable medium having computer executable software code
2 stored thereon, the code for reconstructing an image of a scattering medium, comprising:
3 code to direct energy into the scattering medium at a source location on
4 the scattering medium;
5 code to measure the energy emerging from the scattering medium at a
6 detector location on the scattering medium;
7 code to receive an initial guess of internal properties of the scattering
8 medium;
9 code to predict the energy emerging from the scattering medium using an
10 equation of radiative transfer, wherein the prediction is a function of the initial guess;
11 code to generate an objective function based on a comparison of the
12 prediction with the measurement;
13 code to generate a gradient of the objective function by a method of
14 adjoint differentiation;
15 code to modify the initial guess of the properties of the scattering medium
16 based on the gradient of the objective function; and

17 code to generate an image representation of the internal properties of the
18 scattering medium.

1 60. A computer readable medium having computer executable software code
2 stored thereon, the code for imaging the spatial distribution of optical properties of tissue,
3 comprising:

4 (a) code to direct energy into the scattering medium at a source location on the
5 tissue;

6 (b) code to measure the energy emerging from the scattering medium at a detector
7 location on the tissue;

8 (c) code to receive an initial guess of spatial optical properties of the tissue;

9 (d) code to predict the energy emerging from the tissue using an equation of
10 radiative transfer in an iterative process, wherein the prediction is a function of the initial
11 guess and a refraction index mismatch at a boundary of the tissue, and the iterative
12 process generates a plurality of intermediate predictions;

13 (e) code to generate an objective function based on a normalized comparison of
14 the prediction with the measured energy emerging from the scattering medium;

15 (f) code to generate a gradient of the objective function by adjoint differentiation;

16 (g) code to modify the initial guess of the spatial properties of the tissue based on
17 the gradient of the objective function;

18 (h) code to repeat steps (d) through (g) until at least one of a threshold of
19 modifications to the initial guess is reached and the objective function reaches a
20 threshold; and

- 21 (j) code to generate an image representation of the spatial optical properties of the
- 22 tissue.

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
841
842
843
844
845
846
847
848
849
850
851
852
853
854
855
856
857
858
859
860
861
862
863
864
865
866
867
868
869
870
871
872
873
874
875
876
877
878
879
880
881
882
883
884
885
886
887
888
889
890
891
892
893
894
895
896
897
898
899
900
901
902
903
904
905
906
907
908
909
910
911
912
913
914
915
916
917
918
919
920
921
922
923
924
925
926
927
928
929
930
931
932
933
934
935
936
937
938
939
940
941
942
943
944
945
946
947
948
949
950
951
952
953
954
955
956
957
958
959
960
961
962
963
964
965
966
967
968
969
970
971
972
973
974
975
976
977
978
979
980
981
982
983
984
985
986
987
988
989
990
991
992
993
994
995
996
997
998
999
1000